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OPTIMUM CONTROL OF FLEXIBLE ROBOT ARMS ON FIXED PATHS.

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ABSTRACT

Productivity of the industrial robots are directly related to the speed of the task execution. The speed of the robots can be drastically improved by using better control algorthims and reducing the weight of the manipulator.

The speed of a robotic manipulator is constrained by manipulator dynamics and actuator capabilities. Increasing the size of the actuators is not a solution since that will increase the weight of the the overall system leading to a relatively heavier system.The more realistic approach to the problem is to find the optimum control solution for a manipulator to follow a pre-defined path in minimum time, with limited actuator capabilities.

In terms of the dynamic constraints, the weight of the arms may be the most important factor. If a light-weight arm structure is used, actuators will be able to afford higher speeds during the task execution than they would for rigid arm structure. On the other hand using flexible-arms has a major draw-back which is the flexible vibrations, while increasing the speed.

This paper presents the minimum time control solution of a two link flexible arm with actuator constraints . We solved the minimum time problem with no constraints on the flexible modes and show the time improvement due to the use of light-weight arms. The objective is to modify the trajectory, such that flexible vibrations are bounded while changing the solution from the previous one as little as possible. Practical ways of trajectory modifications for flexible arms are discussed.

INTRODUCTION

Today, most trajectory planning algorithms do not consider the dynamics of the manipulators, rather constant and/or piece wise constant accelerations for the overall task are used and an overall maximum allowable speed is set [5,6,7]. However, robotic manipulators are highly nonlinear dynamic systems, so it is expected that affordable accelerations and decelerations and maximum speeds will vary as a function of states. For the traditional schemes to work, the trajectory must be planned for the worst possible case. The capabilities of the system will be used only a small part of the time. Bobrow et.al. [1] first reported that for every point on the path there is an associated maximum allowable speed and maximum affordable acceleration and

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deceleration, and these values can drastically vary from one state to another. Incorporating the manipulator dynamics into the trajectory planning level, they found the minimum time trajectories for different manipulator models [1,2] with limited actuator capabilities moving along pre-defined paths. Shin and McKay [3] solved the same problem independently.

Light-weight manipulators with the same actuator capabilities will be faster. The main problem associated with the light-weight structures is the flexible vibrations. Fig. 1 conceptually shows the performance improvement in terms of increased speed.

In this paper we show the performance improvements due to

1. use of light-weight arms

2. incorporating the manipulator dynamics into trajectory planning level

3. Discuss flexible vibrations during a minimum time trajectory execution and considerations of path modifications such that flexible vibrations will be bounded. This problem is similar in nature to the one raised by Hollerbach [8].

FLEXIBLE MANIPULATOR DYNAMIC MODEL IN JOINT AND PATH VARIABLES

A general dynamic modelling technique for flexible robotic manipulators was developed by Book using recursive Lagrangian-assumed modes method. Homogeneous transformation matrices are used for kinematic relations of the system [4]. A two link flexible robotic manipulator is modelled using that technique (Fig. 2). In the model no actuator dynamics is considered, rather the net torque input to the links is considered as the input variable. No friction at joints nor in the structural vibrations is considered. Flexibility of each link is approximated with one assumed mode for each link. The dynamic model of the manipulator may be expressed in general terms as :

$$[J]_{4x4} \; \underline{\ddot{q}}=\underline{f}(\underline{q},\underline{\dot{q}}) + \underline{Q} \qquad (2\text{-}1)$$

where

$\underline{q}^T$: $[\theta_1, \theta_2, \delta_1, \delta_2]$ Joint angle and flexible mode time variables

$\underline{Q}^T$: $[T_1, T_2, 0, 0]$ Net input torques

$[J]_{4x4}$: Generalized Inertia Matrix ,symmetric, positive definite.

$\underline{f}^T$: $[f_1, f_2, f_3, f_4]$ Nonlinear dynamic terms including centrifugal gravitational,effective spring and Coriolis.

The problem is to find the minimum time trajectories for a given manipulator with limited actuator capabilities moving along a fixed path, with state constraints (bounded flexible vibration constraint). Once the path to be moved along is specified

$$S=S(x,y) \qquad (2\text{-}2)$$

From inverse kinematic formulation, the corresponding joint angles can be found as

$$\underline{\theta}=\underline{\theta}(s) \quad , \quad \underline{\theta}^T=[\theta_1, \theta_2,] \qquad (2\text{-}3)$$

Similarly, once the speed along the path is known $\dot{S}(S)$

$$\underline{\dot{\theta}}=\underline{\dot{\theta}}\,(s,\dot{s}) \tag{2-4}$$

and

$$\underline{\ddot{\varepsilon}}=\underline{\ddot{\varepsilon}}\,(s,\dot{s},\ddot{s}) \tag{2-5}$$

Knowing the relations (2-3)-(2-5) analytically form or numerically the manipulator dynamics in part can be expressed in path variables.

$$\begin{bmatrix} C_{11}(s,\underline{\varepsilon}) \\ \\ C_{12}(s,\underline{\delta}) \end{bmatrix}_{2x1} \ddot{S} = \begin{bmatrix} T_1 \\ \\ T_2 \end{bmatrix} - \begin{bmatrix} C_{21}(s,\dot{s},\underline{\delta},\underline{\dot{\delta}},\vec{e}_t,\vec{e}_n,\rho) \\ \\ C_{22}(s,\dot{s},\underline{\delta},\underline{\dot{\delta}},\vec{e}_t,\vec{e}_n,\rho) \end{bmatrix} \tag{2-6a}$$

$$\begin{bmatrix} \ddot{\delta}_1 \\ \\ \ddot{\varepsilon}_2 \end{bmatrix} = \begin{bmatrix} J_{33} & J_{34} \\ \\ J_{43} & J_{44} \end{bmatrix}^{-1} \begin{bmatrix} f_3-g_3+h_1(s,\ddot{s},\vec{e}_t) \\ \\ f_4-g_4+h_2(s,\ddot{s},\vec{e}_t) \end{bmatrix} \tag{2-6b}$$

where $f_i=f_i(s,\dot{s},\underline{\delta},\underline{\dot{\varepsilon}})$ (2-7)

$g_i=g_i(s,\dot{s},\vec{e}_t,\vec{e}_n,\rho)$ (2-8)

$J_{ij}=J_{ij}(s,\underline{\dot{\delta}})$ (2-9)

$\vec{e}_t,\vec{e}_n$: Unit tangent and normal vectors along the path.

ρ : Curvature of the path at a point.

Notice that flexible modes also affect the position of the end effector, but are not included in the definition of the path. This is mainly due to the fact that we do not have a "direct" control on the flexible vibrations and would like to keep them as small as possible in general.

FORMULATION OF THE NEAR MINIMUM TIME TRAJECTORY PROBLEM FOR FLEXIBLE MANIPULATORS

Using the classical variational calculus principles, the optimum control/programming problem may be stated as:

$$\text{Minimize } J = \int_0^{t_f} dt = \int_{so}^{s_f} \frac{ds}{\dot{s}} \tag{3-1}$$

$S(So) = So$

$\dot{S}(s_f) = \dot{S}_f$ Initial and final states in path variables.

Subject to :

System dynamics, equations (2-6a) and (2-6b)

Actuator constraints

$$T_{i_{min}}(\underline{\theta},\underline{\dot{\theta}}) \leq T_i \leq T_{i_{max}}(\underline{\theta},\underline{\dot{\theta}}) \qquad i = 1,2 \tag{3-2}$$

Dynamic inequality constraints on flexible modes

$$a_i(t) \leq \delta_i(t) \leq b_i(t) \qquad i=1,2 \tag{3-3}$$

The constraints (3-3) naturally arise in flexible structures. If such a constraint is not imposed there is no guarantee on the accuracy of the end point along the path. At first the problem will be solved without considering these constraints. This solution will be used as a nominal solution for the trajectory modification step so that (3-3) are satisfied.

The solution method we use closely follows Bobrow et.al.'s method with some modifications for flexible manipulators. The solution of the above stated optimization problem follows: for any path $S(x,y)$ with given $\dot{S}_o(S_o),\dot{S}_f(S_f)$ to minimize J, $\dot{S}$ should be as large as possible while satisfying the system dynamics and actuator constraints. In order to do so at any state on the path one should use maximum acceleration or deceleration. Then, the problem is reduced to finding the maximum accelerations and decelerations associated with each state of interest. It can be seen from equation (6a) that for each $(S, \dot{S})$

$$\ddot{S}_d \le \ddot{S} \le \ddot{S}_a \qquad (3\text{-}4)$$

$$\ddot{S}_a = \min \left\{ \ddot{S}_{ai} \right\}$$

$$\ddot{S}_d = \max \left\{ \ddot{S}_{di} \right\}$$

Obviously there may be some range of speeds associated with every point on the path that system can no longer afford to satisfy all conditions (the $\dot{S}$ range that above inequality is violated). Collection of these ranges defines the forbidden region on $(S, \dot{S})$ plane. The boundary between allowed and forbidden regions is constant for a given rigid manipulator for a given task. In the case of flexible manipulators, due to the coupling between equations (6a) and (6b) this boundary is also a function of flexible modes, not only $(S, \dot{S})$. So, depending on the time history of flexible modes and unpredictable disturbances the boundary will vary. This is not true in the rigid case where the true extremum can be found. At this point the problem is to find when to use maximum accelerations and when maximum decelerations (i.e. to find the switching point(s)). See Fig. 3a-3b.

Finding switching points for flexible manipulators:

1. Integrate $\ddot{S}=\ddot{S}(x,y)$ from final state backward in time until it crosses forbidden region or initial position, using maximum deceleration.

2. Integrate $\ddot{S}(x,y)$ Forward in time with maximum acceleration until the boundary is reached or the two curves crossed each other. If the two curve crossed each other before they enter forbidden region, then find that point. This is the last switching point and terminate the search. If not, then

3. Backup on the forward integrated curve and integrate forward with maximum deceleration until a the trajectory passes tangent to the boundary.

4. Then using the point as new starting point go to step two.

Notice that the last switching point is not the exact switching points, because the flexible modes will not match at this point. That will cause one to miss the final state somewhat. Also, when searching for the switching points one has to move in a continuous manner in order to keep track of the flexible mode histories accurately. In that sense, the algorithm given at [1] has been modified for flexible robotic manipulators.

SIMULATION RESULTS AND DISCUSSION

The two-link flexible manipulator model for task one (shown in Fig. 4a) was simulated for the two different cases in order to show the performance improvement achieved due to light-weight system. In both cases actuators have same capabilities. It is found that weight reduction by a factor of 2 results in approximately 60 % time improvements (Fig. 5a and 6a). This improvement, of course, slightly varies depending on the task. Joint actuator histories are shown in Fig. 5b-6c and flexible mode responses are shown in Fig. 5c-6d.

Task 2 (Shown in Fig. 4b) simulated for light-weight manipulator and results are shown Fig 7 a-d. The final trajectory is shown in heavy lines. One interesting point in this simulation is the fact that as soon as the manipulator end point enters the curvature the system must accelerate along the path in order to obey the constraints. In Fig. 5a the curve ab shows that right before the curvature the system is able to afford deceleration (aa' curve), but as end point enters the curvature, then the sudden appearance of a normal acceleration term in the dynamics of the system makes the difference.The other point in the case of flexible arms is that at the last switching point flexible modes are not same,since thay have different histories. This will cause error in the final state reached. See Fig. 6a, 7a. The last switching point needs to be varied from the original result of the above algorithm. This can be done on trial and error basis at the trajectory planning level.

5.CONCLUSION AND FURTHER WORK

In this paper we showed ways to improve performance and productivity of Robotic manipulators With Flexible arms.One way was to use light-weight structures and the other was to incorporate the dynamics of manipulators in to trajectory planning level and make optimum utilization of given manipulator. This method can be used with any path.Application of the method requires manipulator model,Geometric path in work space,and actuator capabilities. Obvoiusly as trajectory gets closer to the forbiden region boundary system capabilities are being used to the limits and any disturbance or uncertainty can easliy put the system into forbiden region .The situation is more dramatic for flexible manipulators.While this analysis is nice in terms of knowing the ultimate capabilities , in practice there will be a saftey factor that will require to keep the optimal trajectory away from the forbideen region certain amount.Research is in progress on the Optimum modification of the trajectories found by above described method so that inequality constraints on the flexible modes will be satisfied.

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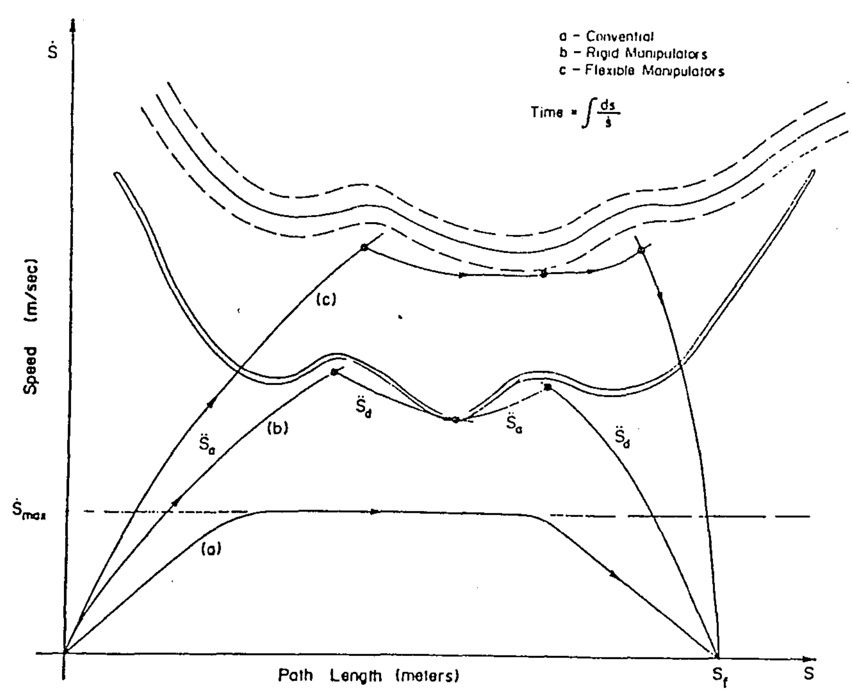

Fig.1 Different trajectory plans.

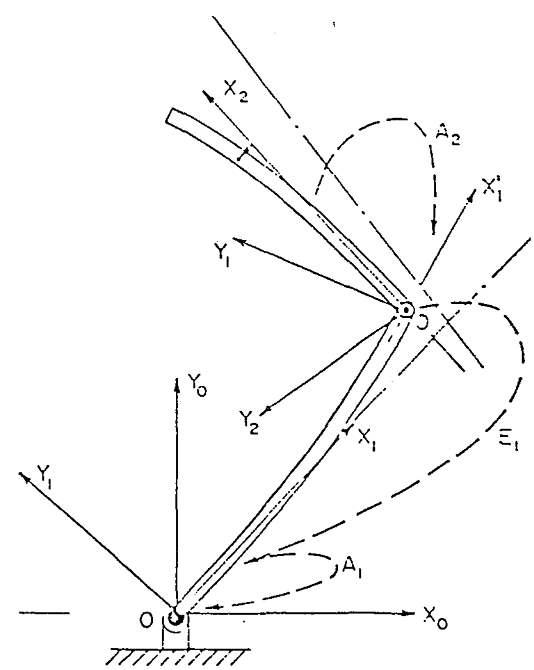

Fig.2 Manipulator Model.

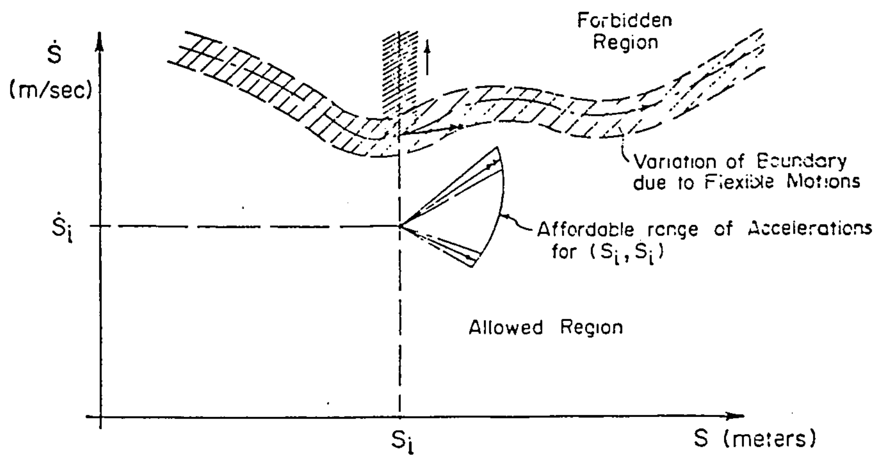

Fig. 3.a (S,S) Plane

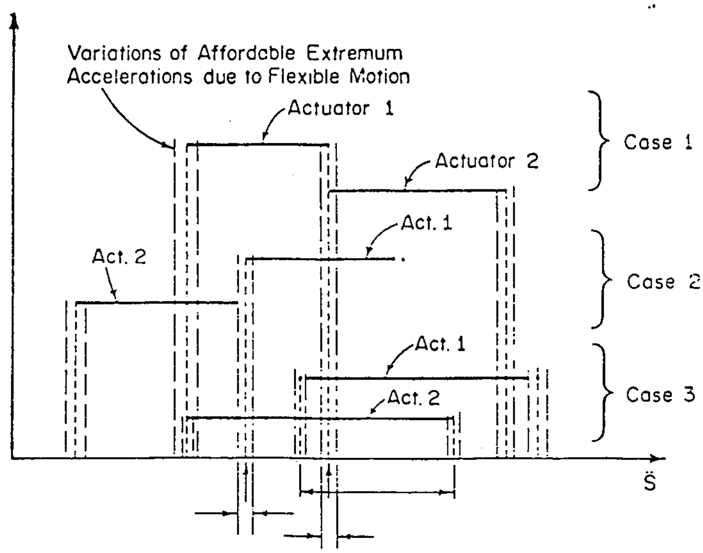

Fig. 3.b Different possible cases during a task.

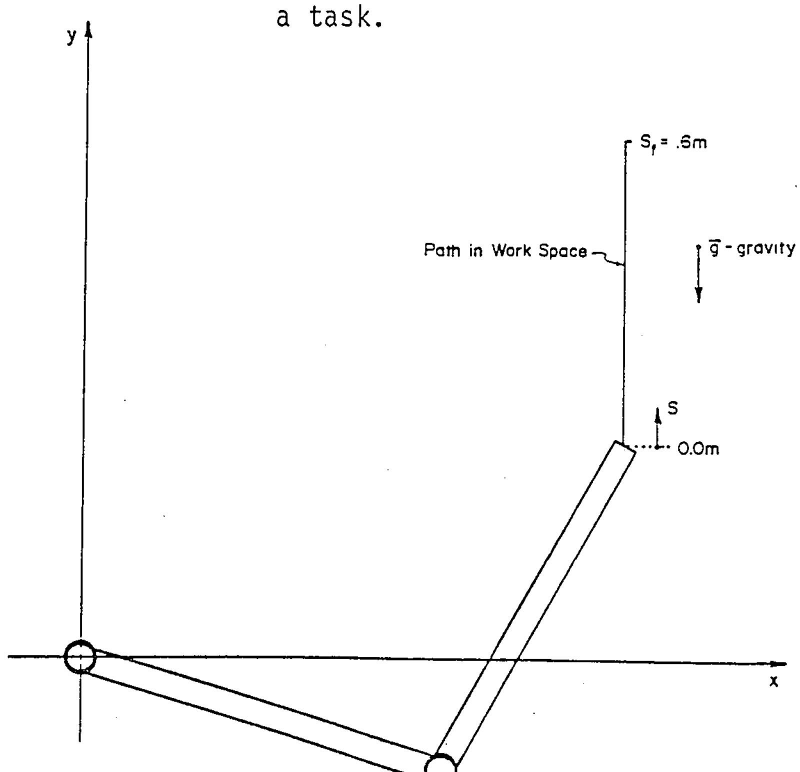

Fig.4.a Task 1 in (x,y) plane.

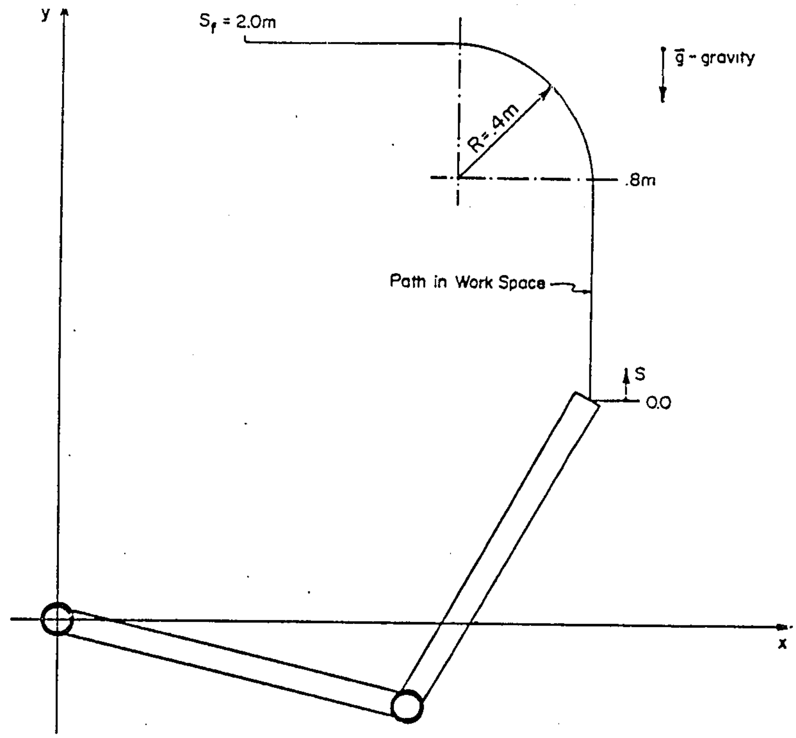

Fig. 4b Task 2 in (x,y) plane.

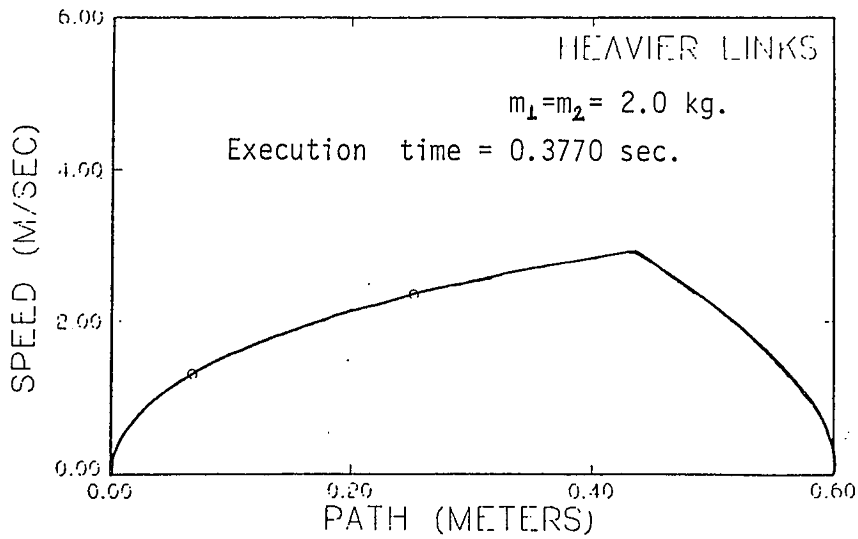

Fig. 5.a Trajectory for Path 1 of heavy links.

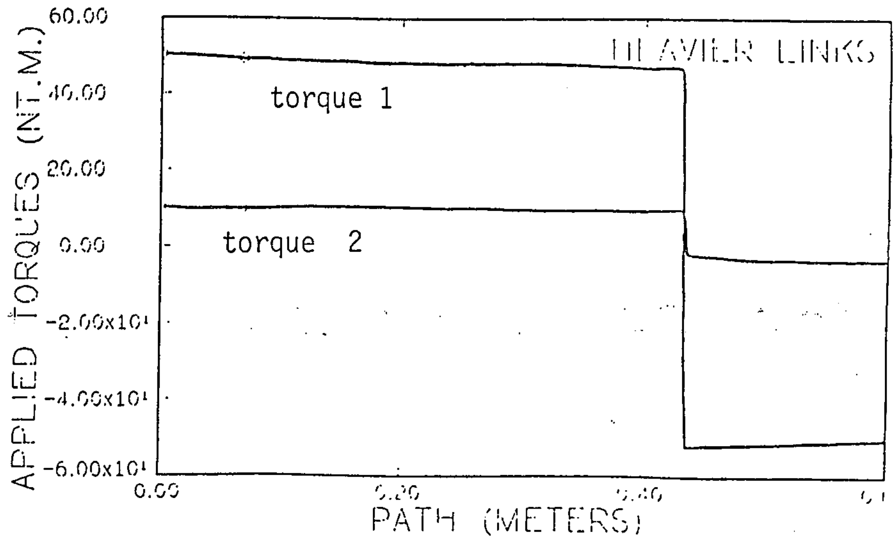

Fig. 5.b Torque histories for path 1.

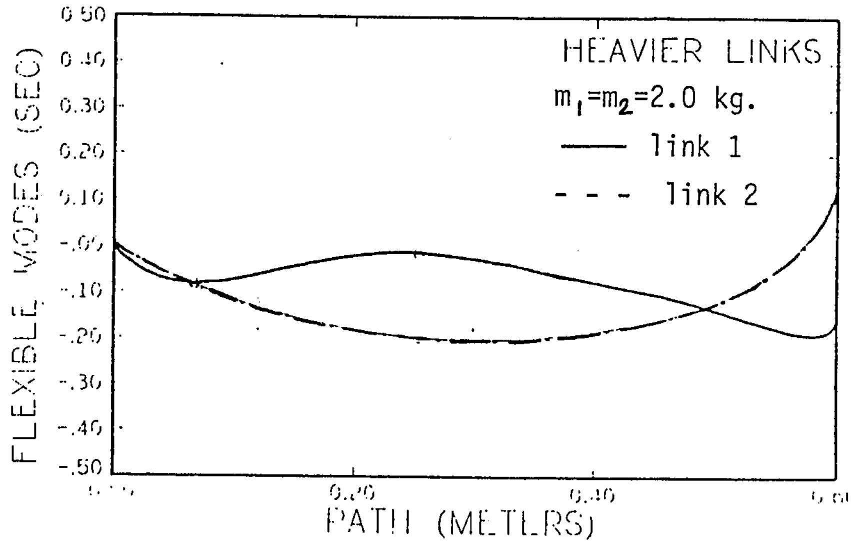

Fig. 5.c Flexible modes time variables

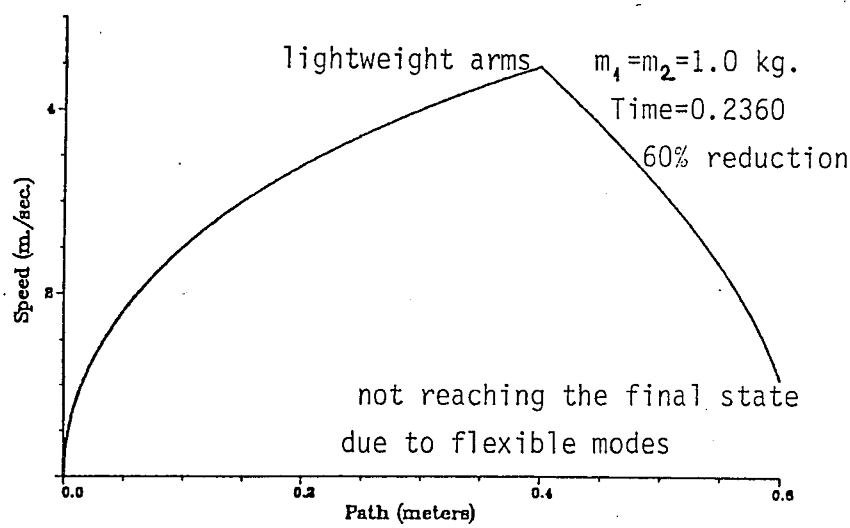

Fig. 6.a Trajectory of lightweight arms.

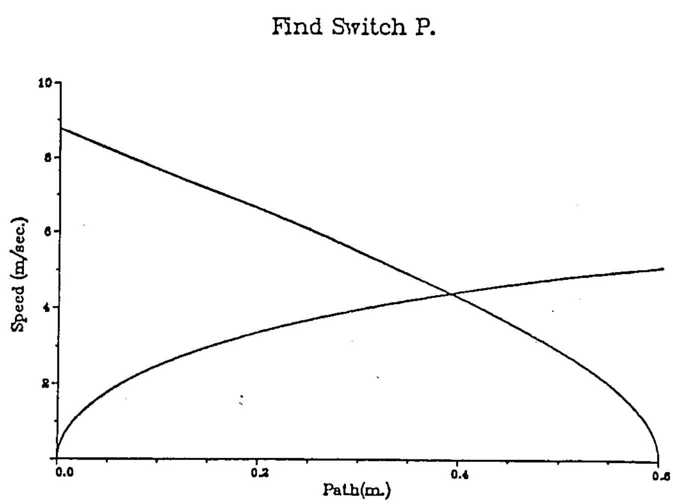

Fig. 6.b Finding the switching point

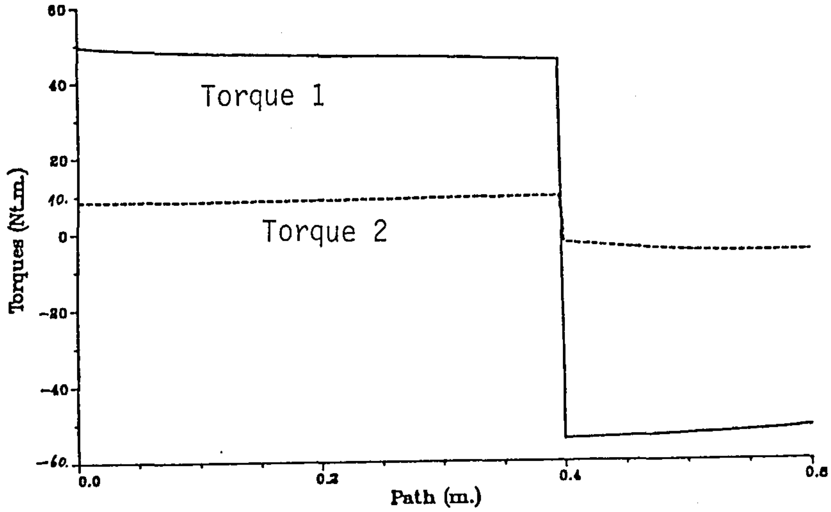

Fig. 6.c Torque histories of lightweight arms along path 1.

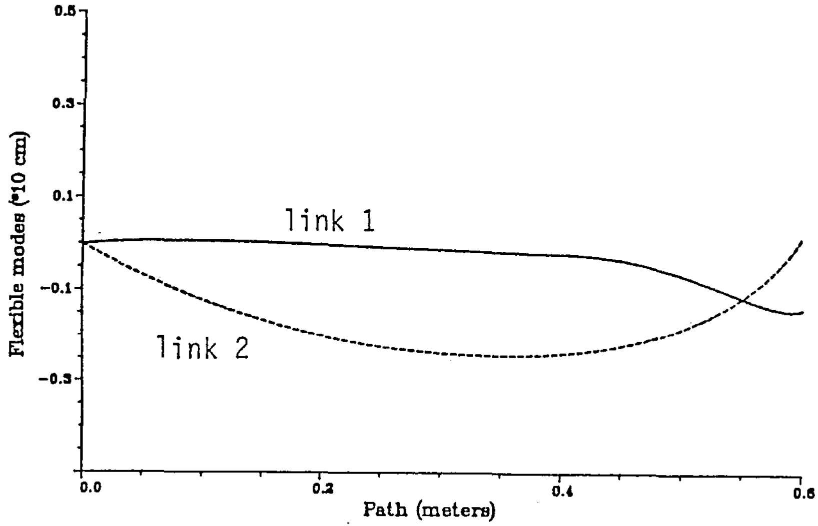

Fig. 6d. Flexible modes along path 1.

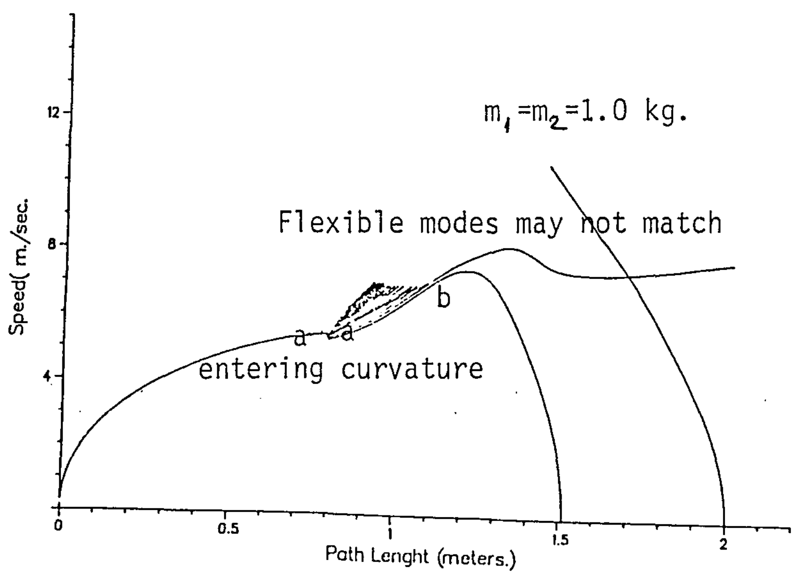

Fig. 7.a Finding the switching points for path 2.

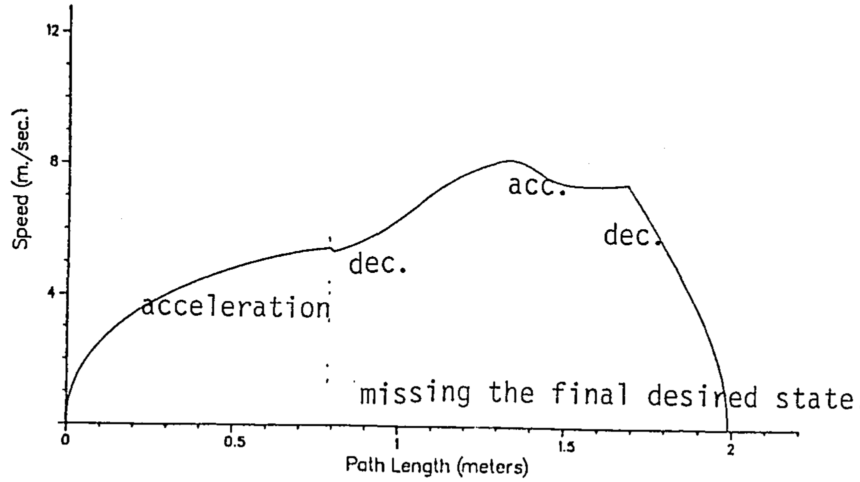

Fig. 7b. Trajectory for path 2.

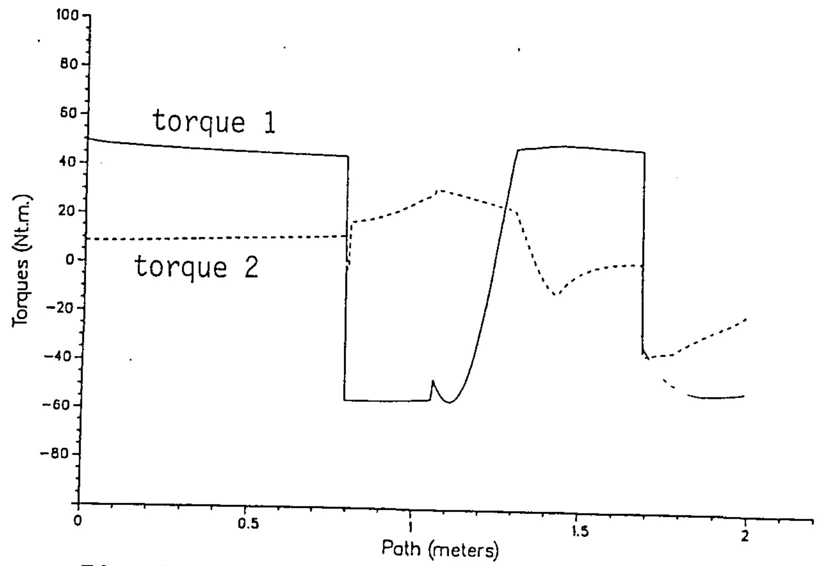

Fig. 7c . Torque histories along path 2

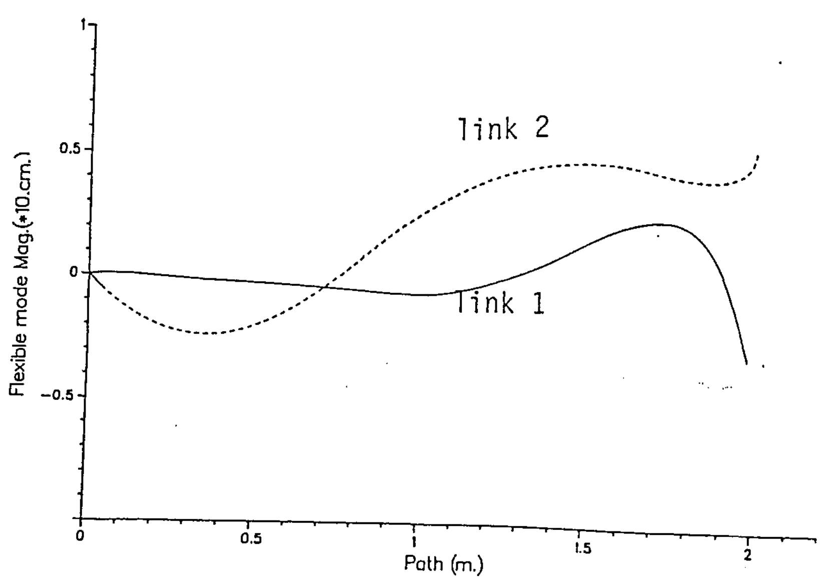

Fig. 7.d Flexible modes along path2.